CSE 417T: Homework 5

Due: April 30 (Friday), 2021

Notes:

- Please submit your homework via Gradescope and check the submission instructions.
- Please download the following files for this homework.
- Homework is due by 11:59 PM on the due date. Remember that you may not use more than 2 late days on this homework, and you only have a budget of 5 in total.
- Please keep in mind the collaboration policy as specified in the course syllabus. If you discuss questions with others you must write their names on your submission, and if you use any outside resources you must reference them. Do not look at each others’ writeups, including code.
- Please comment your code properly.
- There are 5 problems on 2 pages in this homework.

Problems:

1. (40 points) The purpose of this problem is to implement AdaBoost and observe its performance on training and test errors. Please use the same handwritten digit recognition dataset as in homework 4. Again focus on the same two binary classification problems – distinguishing between the digit one and the digit three, and distinguishing between the digit three and the digit five. You need to report the results for both problems (1 versus 3 and 3 versus 5).

   - Code (Complete/submit hw5.py): Use decision stumps learned using information gain as the weak learners. You may use [sklearn.tree.DecisionTreeClassifier](https://scikit-learn.org/stable/modules/generated/sklearn.tree.DecisionTreeClassifier.html) to help learn decision stumps. Read the document carefully to understand how to learn a decision stump from a weighted dataset.
   - Report: For each of the binary classification problems, graphically report the training set error and the test set error as a function of the number of weak hypotheses (from $t = 1$ to 200). You might want to include training/testing error curves in the same plot (i.e., one plot for each classification problem) for easy comparison. Summarize and interpret your results. You should at least comment on the trend of training/test errors and whether AdaBoost is overfitting as the number of weak learners increases.

2. (15 points) (From Russell & Norvig) Construct a support vector machine that computes the XOR function. $\vec{x} = (x_1, x_2)$ denotes the two inputs and $y$ denotes the output. Use $+1$ and $-1$ to represent boolean variables in this question (so the notations are consistent with our lectures). Map the input $(x_1, x_2)$ into a space consisting of $x_1$ and $x_1x_2$. Draw the four input points in this space, and the maximal margin separator. What is the margin? Now draw the separating line back in the original Euclidean input space.

3. (15 points) The key point of the so-called “kernel trick” in SVMs is to learn a classifier that effectively separates the training data in a higher dimensional space without having to explicitly compute the representation $\Phi(\vec{x})$ of every point $\vec{x}$ in the original input space. Instead, all the work is done through the kernel function that computes dot products $K(\vec{x}_i, \vec{x}_j) = \Phi(\vec{x}_i)^T \Phi(\vec{x}_j)$.

For any two points $\vec{x}_i$ and $\vec{x}_j$, show how to compute the squared Euclidean distance of the two points in the projected space, $\Phi(\vec{x}_i)$ and $\Phi(\vec{x}_j)$, without explicitly computing the $\Phi$ mapping. Instead, write down the squared Euclidean distance using the kernel function $K$.

4. (20 points) Construct by hand a neural network with only one hidden layer (of any number of units) that implements XOR( AND$(x_1, x_2), x_3$). You can use sign function as the activation function. Draw your network, and show all weights of each unit. (You may find it useful to first simplify the Boolean formula using common Boolean identities, such as De Morgan’s law.)

5. (10 points) LFD Problem 7.10 (from e-Chapter 7).