

# Mathematical Foundations of Human Computation

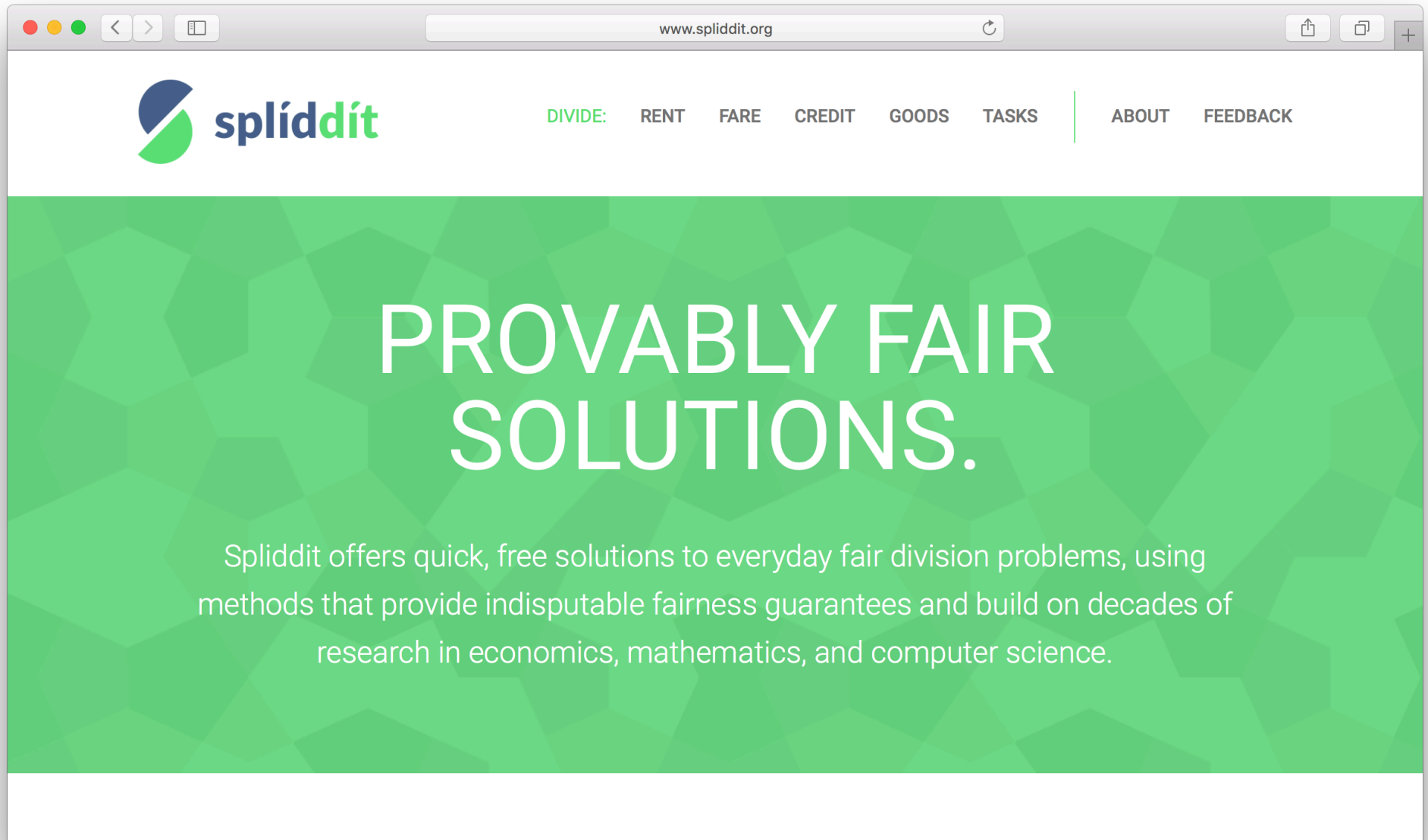
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Why mathematical foundations?

# Mathematical foundations help...

- Formalize desirable properties (e.g., correctness, optimality, scalability, privacy, fairness)
- Predict impact of design decisions (e.g., would quality improve under performance-based pay?)
- Design systems with provable guarantees (e.g., system does not discriminate based on demographic info, data remains private)
- Perform counterfactual analysis (e.g., what would happen if we increased pay by 30%?)


# Warm-Up Example: Fair Division



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- How should the system interact with roommates to extract the value of each room?
  - Build on economics literature on truthfulness
- What makes a set of prices and allocation fair?
  - Envy-freeness: given the prices for each room, every roommate prefers the room he is assigned
  - Pareto-efficiency: no prices/allocation could make a roommate happier without making another less happy
- How do we achieve fair prices and allocation?

# Example: Prediction Markets

 Who will win the 2016 U.S. presidential election?		
Top Predictions		478371 Comments
<b>Clinton</b> 1248	68¢	NC
<b>Trump</b> 2760	32¢	NC

source:  
PredictIt.org

Payoff is \$1 if Clinton wins. If probability of Clinton winning is  $x$ , I should

- Buy at any price less than  $\$x$
- Sell at any price greater than  $\$x$

Market price captures crowd's collective belief

*Chance of Trump  
winning Ohio or  
Pennsylvania?*

Can we generate **coherent prices** (and therefore **coherent predictions**) over large, complex outcome spaces?

# Example: Prediction Markets

- What properties should prices satisfy?
  - Information incorporation
  - No arbitrage
- How to find prices satisfying these properties?
  - Algorithms build on tools from convex optimization
  - Sometimes necessary to relax desired properties
- How should we interpret market prices?
  - Trickier; depends on model of trader behavior



# Example: Performance-Based Pay



1: Nearly every group of animals has its giants, its species which  
2: their fellows as Goliath of Gath stood head and shoulders above  
3: hosts; and while some of these are giants only in comparison  
4: with their fellows, belonging to families whose members are short of  
5: stature, sufficiently great to be called giants under any circumstance  
6: of comparison. Some giants live to-day, some have but recently passed away, and  
7: some lived long ages before man trod this earth. The most gigantic of modern  
8: animals—whales—still survive, and the elephant of to-day suffers but little in  
9: comparison with the mammoth of yesterday; the monstrous Dinosaurs, greatest of  
10: all reptiles—greatest, in fact, of all animals that have walked the  
11: earth—flourished thousands upon thousands of years ago. As for birds, some of  
12: the giants among them are still living, some existed long geologic periods ago,  
13: and a few have so recently vanished from the scene that their memory  
14: lingers amid the haze of tradition. The best known among the extinct  
15: birds, most recent in point of time, are the Moas of New Zealand.  
16: Notice by the Rev. W. Colenso, later on Bishop of New Zealand, of the  
17: missionaries to whom Science is under obligations. Early in 1841  
18: Colenso, while on a missionary visit to the East Cape region, was told  
19: by the natives of Waiapu tales of a monstrous bird, called Moa, which  
20: man, that inhabited the mountain-side some eighty miles away, was  
21: the last of his race, was said to be attended by two equally monstrous  
22: dogs, which kept guard while he slept, and on the approach of man would  
23: immediately rush upon the intruders and trample them to death.  
24: Maoris had seen this bird, but they had seen and somewhat differently described  
25: making parts of their fishing tackle, bones of its extinct relatives, and these  
26: bones they declared to be as large as those of an ox.  
27:  
28: About the same time another missionary, the Rev. Richard Taylor, found a bone  
29: ascribed to the Moa, and met with a very similar tradition among the natives of  
30: a near-by district, only, as the foot of the rainbow moves away as we move  
31: toward it, in his case the bird was said to dwell in quite a different locality  
32: from that given by the natives of East Cape. While, however, the Maoris were

Proofread this text, earn \$0.50

Earn an extra \$0.10 for every  
typo found

performance-based  
payments

# Example: Performance-Based Pay

- Goals: Use theoretical tools to...
  - Predict the impact of payments on worker quality (a form of counterfactual analysis)
  - Design performance-based payments to optimally trade off cost and benefit (a learning problem)
- Both require a model of worker behavior

# Example: Performance-Based Pay

- Initial theory derived under standard econ model, worker chooses to produce work of the quality  $q$  that maximizes her expected utility:

$$\begin{aligned} & \text{BasePayment} \\ & + \text{BonusPayment} \times \text{Pr}(\text{GetBonus} \mid q) \\ & - \text{Cost}(q) \end{aligned}$$

probability of receiving the bonus

intrinsic cost of performing the work

- Algorithm designed to optimize worker payments adaptively

# Example: Performance-Based Pay

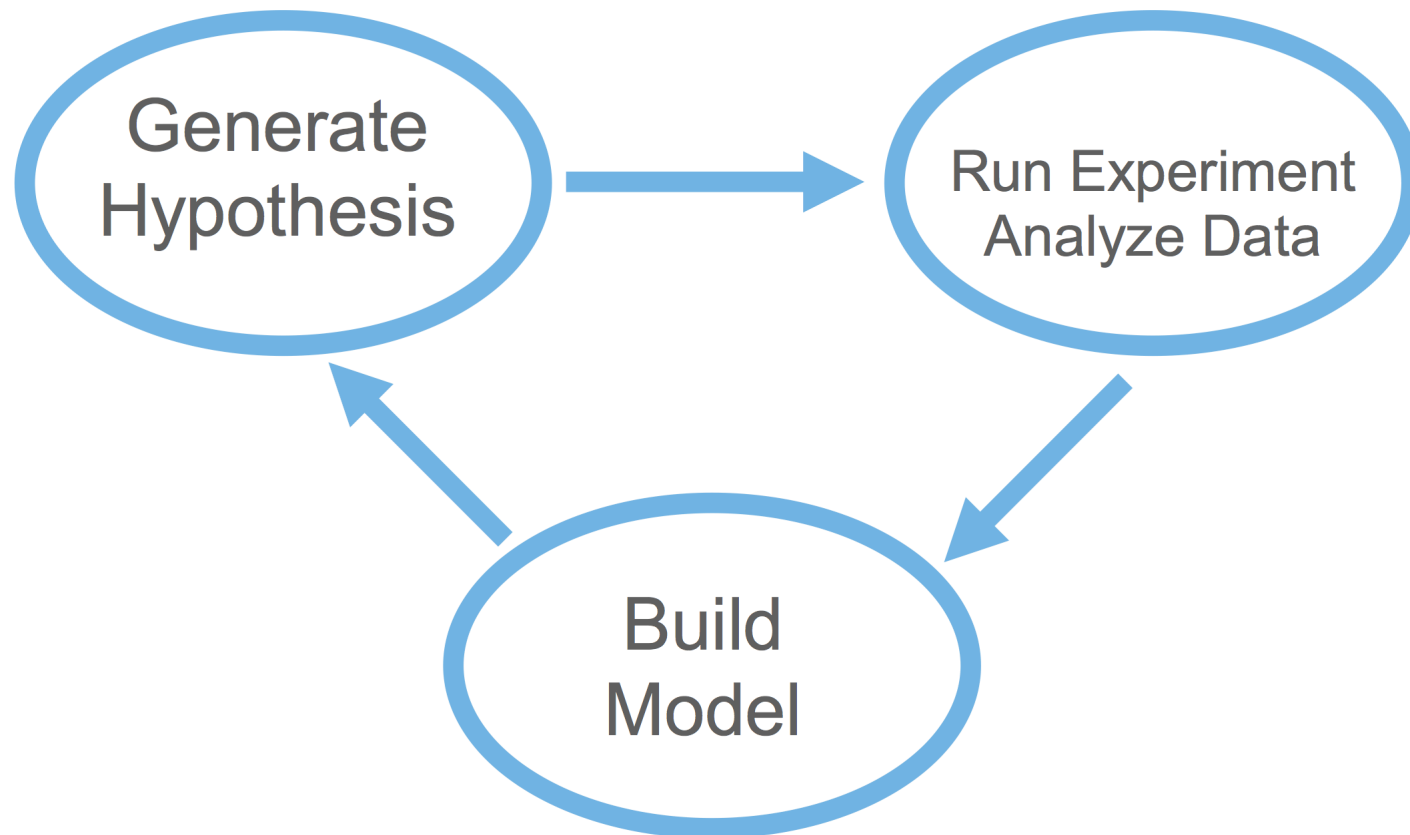
- Experiments showed a small tweak to this model better explains observed worker behavior:

$$\begin{aligned} & \text{BasePayment} \times \text{Pr}(\text{GetBase} \mid q) \\ & + \text{BonusPayment} \times \text{Pr}(\text{GetBonus} \mid q) \\ & - \text{Cost}(q) \end{aligned}$$

*subjective probability of receiving the base*

*subjective probability of receiving the bonus*

# Challenge 1: How to design models that accurately incorporate human behavior



[source: Sid Suri]

Challenge 2: How to foster dialog between theoretical, experimental, and empirical research & across disciplinary boundaries

Challenge 3: How to get results that  
generalize beyond inherently  
mathematical problems

## Challenge 4: How to handle issues of transparency, interpretability, and ethical implications



Welcome again!